Document made available under the Patent Cooperation Treaty (PCT)

International application number: PCT/NZ04/000331

International filing date: 23 December 2004 (23.12.2004)

Document type: Certified copy of priority document

Document details: Country/Office: NZ
Number: 530377
Filing date: 24 December 2003 (24.12.2003)

Date of receipt at the International Bureau: 17 January 2005 (17.01.2005)

Remark: Priority document submitted or transmitted to the International Bureau in compliance with Rule 17.1(a) or (b)

PCT/NZ2004/000331

CERTIFICATE

This certificate is issued in support of an application for Patent registration in a country outside New Zealand pursuant to the Patents Act 1953 and the Regulations thereunder.

I hereby certify that annexed is a true copy of the Provisional Specification as filed on 24 December 2003 with an application for Letters Patent number 530377 made by JOHN REDMAYNE.

Dated 12 January 2005.

Neville Harris
Commissioner of Patents, Trade Marks and Designs

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530377

CONVERTED INTO A PROVISIONAL SPECIFICATION

PATENTS FORM 5	Number
PATENTS ACT 1953	Dated
COMPLETE SPECIFICATION	

SYSTEM AND METHOD FOR PRICING SECURITIES

I, **JOHN REDMAYNE, a New Zealand citizen, of Penny Road, RD 9, Palmerston North, New Zealand**, do hereby declare the invention for which I pray that a patent may be granted to me, and the method by which it is to be performed, to be particularly described in and by the following statement.

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SYSTEM AND METHOD FOR PRICING SECURITIES

FIELD OF THE INVENTION

The invention relates generally to the processing of financial data. More particularly, the invention relates to a technique for modelling the price of securities issued by or referenced on a firm, or given the prices observed for such securities, a technique for modelling the expected risk, return and other characteristics of those securities, including (without limitation) the probability of default, the loss given default and the expected default loss.

BACKGROUND OF THE INVENTION

The debt and equity securities issued by a firm can be viewed as contingent claims over the assets of the firm, with the value of those claims being contingent on the potential future value of the firm. In particular the common equity holders in a firm can be viewed as having a call option over the firm's assets, exercisable by paying off the firm's outstanding debt. This option-theoretic view of the firm was first noted by Black and Scholes (in Black, F. and Scholes, M. The Pricing of Options and Corporate Liabilities. Journal of Political Economy, 1973 May/June, 81, 637-654).

Merton (in Merton, R.C. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. Journal of Finance, 1974 May, 29 (2), 449-470) went on to further analyse this option-theoretic view of the firm, and to develop a model for predicting or explaining the credit spread on a firm's bonds (the credit spread is the difference between the promised yield on the firm's bonds and the yield on a risk free bond, such as a government bond, of equivalent maturity). This model, commonly referred to as the "Merton model" or the "structural model", is also used to predict the probability of a firm defaulting on its debt.

The Merton model has been found to be unable to properly explain the observed credit spreads on firms' bonds. Researchers have sought to improve the performance of the

Merton model through various refinements, but with limited success. Investors in credit risky securities, banks, regulators and others all have an interest in models that can better explain the pricing of credit risky securities and in understanding the risk, return and correlation attributes of those securities.

Tests of the Merton model have found that it tends to underestimate the credit spread on firms' bonds, or that an excessive input for the volatility of the firm's assets is required in order to fit the model. The reason for this is that the Merton model only provides an estimate of the expected default loss (being the expected probability of default times the expected loss given default) on a firm's bonds in a risk neutral world. Where a real world expected rate of return on a firm's assets is used in the Merton model, instead of the risk free rate, together with a realistic input for the volatility of the firm's assets, the credit spreads predicted by the Merton model tend to be even lower.

A commercial application of the Merton model is to use it to estimate the probability of firms defaulting on their debt obligations. However, the default probabilities produced by the model have been found to only contain useful relative, as opposed to absolute, information on expected default.

Recent analysis of the components of the credit spread on firms' bonds identifies that the expected default loss represents only part of the total observed credit spread on corporate bonds. Factors explaining the difference between the credit spread and expected default loss on a firm's bonds are likely to include tax effects, a liquidity premium and a risk premium. The Merton model does not allow for these factors. In particular, the Merton model does not allow for a risk premium within the expected return on a firm's bonds – all of the credit spread on a firm's bonds is treated as being the expected default loss.

Hsia (in Hsia, C.C. Estimating a Firm's Cost of Capital: An Option Pricing Approach. Journal of Business Finance & Accounting, 1991 January, 18 (2), 281-287) derived a method for estimating the expected rate of return on a firm's equity utilising the Black-Scholes-Merton approach. Under Hsia's method the firm's capital structure is stylised

as a single class of equity and a single zero coupon bond. Analysis of Hsia's method shows that Hsia's expected excess rate of return (being the expected rate of return in excess of the risk free rate of return) on equity is equal to the Merton-model-derived expected default loss times the instantaneous volatility of the firm's equity divided by the instantaneous volatility of the firm's debt, all in a risk neutral world. Hsia's method is flawed as there is no reason why the expected excess rate of return on the firm's equity should be proportional to the expected default loss on the firm's debt.

Other models of the firm allow for a risk premium on credit-sensitive securities issued by or referenced on the firm, for example Giesecke and Goldberg (in Giesecke, K. and Goldberg, L.R. The Market Price of Credit Risk. Working paper, draft dated 20 November 2003, downloaded 28 November 2003 from http://www.orie.cornell.edu/~giesecke/mpocr.pdf). They measure risk in this context in terms of diffusive price volatility and in their proposed model the market price of risk is given by the excess return on the firm per unit of firm risk. Where risk is measured as the standard deviation of expected returns, the excess return per unit of risk is often referred to by experts in the art as the "Sharpe ratio". A shortcoming with models of this type is that once the firm issues more than one class of security it is possible, for each security, for the excess return per unit of that security's risk to exceed the excess return on the firm per unit of firm risk. In other words a range of possible firm capital structures may exist whereby the Sharpe ratio for each of the firm's securities may exceed the Sharpe ratio for the unlevered firm.

SUMMARY OF INVENTION

The broad concept of the invention is to decompose the promised yield spread on a firm's debt securities into an expected default loss component and an expected risk premium. The expected risk premia on two or more of the firm's securities, which may include the firm's common equity, (or be other securities referenced to the firm) are related to each other by designating that for each priced risk factor the price per unit of risk is the same for all securities. In the case of the volatility of expected returns (and

any higher statistical moments thereof), the price of each such risk factor is firm specific.

One embodiment of the invention is to incorporate the risk premia and risk premia relationship into an option-theoretic model of the firm. In another embodiment information on the volatility and correlation of two or more securities issued by, or referenced to, the firm is processed to generate estimates of the expected default loss and the expected rate of return on the said securities.

DETAILED DESCRIPTION OF INVENTION

The system and method comprises a number of steps carried out by a computer program. Several of these steps are novel and their application results in commercially useful models. Particular embodiments of the invention are also provided.

The steps of a preferred form of the invention are as follows:

1. Specify a return process for the value of the firm's assets, using real world (as opposed to "risk neutral world") parameters. The return process can include a defined statistical distribution (e.g. the lognormal return distribution often used in financial models), be based on an empirical distribution or such other process specified by the user. In statistical terms the distribution of asset returns from the specified process may include user defined attributes such as jumps, skewness and kurtosis. Furthermore, the return process may be a function of several factors or processes, including, for example, stochastic interest rates and/or allowance for taxes.

2. Specify the pay-off structure for the securities issued by the firm and for any other potential claims on or referenced to the firm's assets, as specified by the user. Without loss of generality the securities or potential claims can include common equity (being the residual claim over the firm's assets), one or more classes of debt security, taxes, bankruptcy costs, firm value lost upon default etc. The pay-off

structure is related to a default barrier, which may be specified by the user or be endogenous to the model. The pay-off, or recovery upon default, of default risky securities issued by the firm may be specified by the user or be endogenous to the model.

3. Given 1. and 2. above the following can be defined (for example, depending upon the complexity of the model, by formula):

   (a) The expected mean pay-off of each security issued against or referenced to, or claim on the firm's assets;

   (b) The expected volatility of each security issued against or referenced to, or claim on the firm's assets;

   (c) Any higher statistical moments or statistical attributes of each security issued against or referenced to, or claim on the firm's assets (for example; skewness, coskewness, kurtosis, cokurtosis, jump risk);

   (d) The correlation of expected returns between pair of each securities issued against and/or referenced to, and/or claims on the firm's assets;

   (e) The correlation of expected returns between each security issued against or referenced to, or claims on the firm's assets and the expected returns on the firm's assets.

4. The value of each security (or claim over the firm's assets) at the beginning of the period of interest is related to the value of that claim/security at the end of the period of interest by a discount rate (or expected rate of return) specific to that claim/security. And the value of the firm's assets at the beginning of the period of interest is related to the value of the firm's assets at the end of the period of interest by a discount rate (or expected rate of return) specific to the firm's assets.

5. The discount rate for each of the firm's securities comprises the sum of a risk free rate of return and a risk premium for each risk factor that is being priced in the

model (for example, in a basic implementation the only priced risk factor would be the volatility, i.e. variance, of expected returns). For each security each risk premium (for each priced risk factor) is the product of the security's exposure or sensitivity to that risk factor and a price for that risk factor. In the case of the volatility of expected returns (and any higher statistical moments thereof), the price of each such risk factor is firm specific.

In its simplest form only one risk factor is priced, the volatility of each security's expected returns arising from the diffusive price risk of the firm's assets. In more complex specifications of the model one or more additional risk factors are priced, including (but not limited to); skewness, kurtosis, other higher statistical moments, jumps, interest rate risk factors, liquidity and size.

The firm specific price for each risk factor related to the moments of expected return (e.g. volatility) is the <u>same for each security</u> issued by the firm, but <u>not necessarily the same</u> as for the price for each such risk factor when <u>measured across the firm's total assets</u>. The model of the invention differs from the Giesecke and Goldberg model as the latter specifies that the firm specific price for the diffusive price volatility risk factor, used to value each security issued by the firm, <u>is the same</u> as the price for the diffusive price volatility risk factor when measured across the firm's total assets.

The price per unit of diffusive volatility risk in a Merton-based model is generally greater for individual securities than it is for the firm's assets in total. Hence equating this price of risk across all of the firm's securities, as opposed to equating it to the price of risk for the firm's assets in total, better enables the model to be fitted to real world data.

The invention also differs from other models known in the art, such as the Capital Asset Pricing Model and the Arbitrage Pricing Theory, which latter models include a market-wide price of risk. The latter models are not based on firm specific measures of total risk, but rather are implemented by only pricing the systematic or market correlated element of firm risk.

6. Given the framework described above, the model can be solved for one or more unknown parameters using standard mathematical tools. The fundamental valuation principle in 4. and 5. above provides q conditions the model must satisfy for $q+1$ types of security the firm has on issue (including equity). If more than q model input parameters are unknown, then one manner in which the unknown input parameters can be solved is by specifying additional conditions, such that one or more of the statistical attributes of the model set out in 3.(b) to 3.(e) above are set to equal observed or expected real world values provided by the user for those statistical attributes.

In a simple example application of the model of the invention the firm is assumed to have only a single class of debt on issue, being zero coupon debt all maturing on the same date and ranking equally on liquidation of the firm. The firm is assumed to pay no dividends and the other assumptions conventionally made by those skilled in the art are made (e.g. there are no taxes, no transaction costs etc.).

Defining:

S_n is the value of the (common) equity of the firm at time n

V_n is the value of the firm (the underlying assets) at time n. In this example the value of the firm is the sum of the values of the firm's debt (B) and equity (S)

X is the face value of the firm's debt (B), which is assumed to be a single zero-coupon bond, at maturity (i.e. the "exercise price")

T is the time to maturity (the expiry date) of the firm's debt in years

r_V is the rate of return on the firm's assets, per annum

r_S is the rate of return on the firm's equity, per annum

r_B is the rate of return on the firm's debt, per annum

y is the promised yield on the firm's debt, per annum

$$d_1 = \left(\left[\ln\left(\frac{V_0}{X}\right) + r_V T\right] \Big/ \sigma_V \sqrt{T}\right) + (1/2)\left(\sigma_V \sqrt{T}\right)$$

$$d_2 = d_1 - \sigma_V \sqrt{T}$$

$N()$ is the cumulative probability of the standard normal distribution with d_1 or d_2 as the upper limit

r is the risk free rate of return per annum

σ_V is the standard deviation of rates of return on the firm's assets per annum

σ_B is the standard deviation of rates of return on the firm's debt per annum

σ_S is the standard deviation of rates of return on the firm's equity per annum

ρ_{jk} is the correlation coefficient of the two variables *j* and *k*.

Then the value of the equity of the firm (*S*) can be viewed as the value of a call option on the firm's assets (*V*), exercisable at time *T* by paying off the face value of debt outstanding at that time (*X*).

At time *T* the expected values of the firm's assets (V_T), the firm's debt (B_T) and the firm's equity (S_T) are given as:

$$V_T = V_0 e^{r_V T} \tag{1}$$

$$B_T = V_T[1 - N(d_1)] + XN(d_2) = B_0 e^{r_B T} \tag{2}$$

$$S_T = V_T N(d_1) - XN(d_2) = S_0 e^{r_S T} \tag{3}$$

And at time *T* the promised amount of the firm's debt (*X*) is given as:

$$X = B_0 e^{yT} \tag{4}$$

Substituting equations (1) and (4) into equations (2) and (3) and rearranging results in:

$$r_B = \ln\left(\frac{V_0 e^{r_V T}[1 - N(d_1)] + B_0 e^{yT} N(d_2)}{B_0}\right) \Big/ T \qquad (5)$$

$$r_S = \ln\left(\frac{V_0 e^{r_V T} N(d_1) - B_0 e^{yT} N(d_2)}{S_0}\right) \Big/ T \qquad (6)$$

The excess rate of return for a security is defined as the expected rate of return in excess of the risk free rate of return. Under the model of the invention the excess rate of return for each security issued by the firm is proportional to the riskiness of that security. There may be more than one risk factor that is priced in the case of one, some or all of the securities issued by the firm. However, the price per unit of risk (λ_m), for each priced risk factor (m), is the same for all securities issued by the firm.

In this example only one risk factor is priced, being the volatility (i.e. the standard deviation, σ) of the expected returns for each security. Hence the following condition, in this example, is specified in the model of the invention:

$$\lambda_\sigma = \frac{r_B - r}{\sigma_B} = \frac{r_S - r}{\sigma_S} \qquad (7)$$

σ_B and σ_S can be specified consistent with the model, for example instantaneously or over a discrete time period, such as T.

Combining equations (5) and (6) into (7) provides the following equation for pricing (or estimating the expected returns of), in this example, the firm's debt and equity:

$$\frac{\ln\left(\frac{V_0 e^{r_V T}[1 - N(d_1)] + B_0 e^{yT} N(d_2)}{B_0}\right) \Big/ T - r}{\sigma_B} = \frac{\ln\left(\frac{V_0 e^{r_V T} N(d_1) - B_0 e^{yT} N(d_2)}{S_0}\right) \Big/ T - r}{\sigma_S} \qquad (8)$$

The difference between the promised yield on a firm's debt and the risk free rate of return available for an equivalent maturity is the "credit spread" ($= y - r$). Under the model of the invention only a proportion of the credit spread is attributed to the expected default loss being the product of the probability of the firm being in default at time T and the loss

given default. That proportion is equal to $y - r_B$. The remainder of the credit spread, in this simple example, is a risk premium to reward an investor in the firm's debt for the volatility of the expected returns from that debt, in this case equal to $r_B - r$.

Under the model of the invention the following condition is not specified and need not hold true after equation (7) is satisfied:

$$\lambda_\sigma = \frac{r_V - r}{\sigma_V} \tag{9}$$

Equation (9) specifies that the price of variance or volatility risk for the firm is given by the excess return on the firm per unit of firm risk, as proposed by Giesecke and Goldberg (2003).

A further embodiment of the invention utilises a novel and original method for using information on the volatility and correlation of security returns to relate or determine the expected default loss on debt securities issued by, or referenced to, the firm and the expected returns on any securities issued by, or referenced to, the firm. Under this embodiment different input parameters are used, but the same principles for the price of risk are applied. In particular the firm specific price for each risk factor related to the moments of expected return (e.g. volatility) is the <u>same for each security</u> issued by the firm. In addition this embodiment of the invention allows the expected default loss on a debt security to be estimated more directly.

The steps of a preferred form of this embodiment of the invention are as follows:

1. Relate the volatility and correlation of returns of securities issued by, or referenced to, the firm to the expected default loss of one of the said securities (security j), wherein the annualised expected default loss on security j (EDL_j) is designated as:

$$EDL_j = \ln\left(\rho_{jk}\sqrt{\left(e^{\sigma_j^2 T}-1\right)\left(e^{\sigma_k^2 T}-1\right)}+1\right)\Big/T \qquad (10)$$

where:

j is the class of the firm's debt or similar security issued by, or referenced to, the firm for which the expected default loss is being estimated

k is the class or classes of security issued by, or referenced to, the firm that rank immediately behind security j in terms of priority upon a liquidation or default event

T is the time horizon of interest to the user in years

σ_j is the standard deviation of rates of return, per annum, on the class of the firm's debt, or similar security issued by, or referenced to, the firm for which the expected default loss is being estimated

σ_k is the standard deviation of rates of return, per annum, on the portfolio comprising the class or classes of security issued by, or referenced to, the firm that are next most junior to j

ρ_{jk} is the correlation coefficient of the rates of return for j and k.

2. Determine the annualised expected rate of return on security j (r_j) by reference to the annualised promised yield on said security (y_j) and the annualised expected default loss on said security, as designated above, where:

$$r_j = y_j - EDL_j \qquad (11)$$

3. The firm specific price of volatility risk (λ_σ) is then designated using the above parameters and the risk free rate of return (r) as follows:

$$\lambda_\sigma = \frac{r_j - r}{\sigma_j} \qquad (12)$$

4. The expected rate of return on security class or classes k (r_k) is then designated as follows:

$$r_k = r + \lambda_\sigma \sigma_k \qquad (13)$$

5. Where security class or classes k are debt securities, the expected default loss on said securities is then designated by combining the promised yield on said securities (y_k) and their expected return (r_k) as follows:

$$EDL_k = y_k - r_k \tag{14}$$

Example Methods for Fitting One Embodiment of the Model of the Invention With More Than One Unknown Input Variable

In the example given above of the first embodiment of the invention, where $V = S + B$, equation (8) can be solved (where a mathematically feasible solution exists) where one of the input variables is unknown. This can be done using standard mathematical tools.

Where more than one input variable is unknown more than one feasible solution may exist, unless additional conditions are imposed on the model. Typically an additional condition is imposed for each additional unknown input variable. Additional conditions that can be imposed are to match observed or otherwise estimated (outside of the model) values for various parameters, as provided by the user, with model implied values for those same parameters. In this simple example model implied values for the parameters σ_B, σ_S and / or ρ_{BS} can be estimated and the model solved to equate these with "known" estimates for the same parameters.

Furthermore, for example, if the model (in the form illustrated in this example) is being solved or implemented in such a way that a "known" time series of firm values is available then model implied values for the parameters σ_V, ρ_{VS} and / or ρ_{VB} can also be estimated and the model solved to equate these with "known" estimates for the same parameters.

Example Calculation of Variances and Correlations

Under the model of the invention, in the above example, the <u>instantaneous</u> volatility of the firm's debt and equity returns are given by the following formulae:

$$\sigma_B = \sigma_V \frac{V_0}{B_0} e^{(r_V - r_B)T} [1 - N(d_1)] \tag{15}$$

$$\sigma_S = \sigma_V \frac{V_0}{S_0} e^{(r_V - r_S)T} N(d_1) \tag{16}$$

It is noted that equations with some similarity to equations (15) and (16) are known in the art in the context of the Merton model. The equation similar to (16) that is known in the art, and that is sometimes used as an additional condition to solve the Merton model where two input variables are unknown, is:

$$\sigma_S = \sigma_V \frac{V_0}{S_0} N(d_1) \tag{17}$$

Under the model of the invention, in the above example, the <u>discrete period</u> volatility of the firm's debt and equity returns (measured over period T) and the pair-wise correlations between the two securities and the firm value are given by the following formulae:

$$\sigma_B = \sqrt{\ln\left(\frac{V_T^2[1 - N(d_3)]e^{\sigma_V^2 T} + X^2 N(d_2)}{B_T^2}\right) \Big/ T} \tag{18}$$

$$\sigma_S = \sqrt{\ln\left(\frac{V_T^2 N(d_3) e^{\sigma_V^2 T} - 2V_T X N(d_1) + X^2 N(d_2)}{S_T^2}\right) \Big/ T} \tag{19}$$

$$\rho_{BS} = \frac{X - B_T}{B_T \sqrt{\left(e^{\sigma_S^2 T} - 1\right)\left(e^{\sigma_B^2 T} - 1\right)}} \tag{20}$$

$$\rho_{VB} = \frac{V_T[1 - N(d_3)]e^{\sigma_V^2 T} + XN(d_1) - B_T}{B_T \sqrt{\left(e^{\sigma_V^2 T} - 1\right)\left(e^{\sigma_B^2 T} - 1\right)}} \tag{21}$$

$$\rho_{VS} = \frac{V_T N(d_3) e^{\sigma_V^2 T} - XN(d_1) - S_T}{S_T \sqrt{\left(e^{\sigma_V^2 T} - 1\right)\left(e^{\sigma_S^2 T} - 1\right)}} \tag{22}$$

Where the additional term, not previously defined, is:

$$d_3 = d_1 + \sigma_V \sqrt{T} \tag{23}$$

The invention provides a framework for calculating the:

1. expected return on debt and equity securities
2. expected volatility on the expected return on debt and equity securities
3. expected correlation between pairs of securities issued by the same firm
4. expected correlation between each security issued by the firm and the firm's total value
5. value of debt and equity securities
6. real world probability of a firm defaulting
7. expected recovery on securities in default
8. optimal capital structure of a firm
9. yield at which a firm can be expected to issue debt securities.

In the model of the invention the firm specific price for priced risk factors relating to the volatility (and any higher moments that are priced) of expected returns is the same for two or more securities issued by the firm, but not necessarily the same as for the price for such risk factors when measured across the firm's total assets.

The model of the invention provides novel and original methods for fitting the family of Merton-based models by equating model values to observed/expected real world values for:

(a) Volatility of the expected returns on non-equity securities (it is known in the art how to do this for the instantaneous volatility of expected equity returns).

(b) Skewness, kurtosis and higher moments of the expected returns on securities.

(c) Correlation between pairs of securities.

(d) Correlation between a security and the firm's total assets.

The model of the invention also provides novel and original formulations for the family of Merton-based models (in a simplified application where the firm has only issued equity and one class of zero coupon debt security) for:

(a) Instantaneous volatility of expected returns for debt and equity securities in a risk premium inclusive Merton-based model

(b) Discrete period volatility of expected returns for debt securities

(c) Discrete period correlation between pairs of expected returns for securities.

(d) Discrete period correlation between expected returns for each security and the firm's assets.

Accordingly, although the invention has been described in detail with reference to particular preferred embodiments, persons possessing ordinary skill in the art to which this invention pertains will appreciate that various modifications, changes and enhancements may be made without departing from the spirit and scope of the claims that follow.

CLAIMS:

1. A method of calculating the expected rate of return for a security issued by and/or referenced on a firm comprising the steps of:

calculating the expected future value of the assets of the firm (V_T);

calculating the future promised amount of the debt of the firm (X);

calculating the expected future value of each class (there being *n* classes) of debt issued by the firm (B_{nT});

calculating the expected future value of the common equity of the firm (S_T);

calculating the exposure of each security to each priced risk factor (*m*);

calculating a price per unit of risk (λ_m) for each priced risk factor (*m*) in which λ_m is the same for two or more securities issued by the firm

designating that all λ_m's that relate to the volatility (or other higher moments) of the expected rate of return on securities are specific to the firm;

calculating the excess rate of return for the security based at least partly on its expected exposure to each priced risk factor and the λ_m's;

calculating the expected rate of return on each class of debt issued by the firm (r_{nB}) based at least partly on the value of each class of debt at the present time (B_{n0}) and B_{nT}; and

calculating the expected rate of return on the common equity of the firm (r_S) based at least partly on the value of the common equity at the present time (S_0) and S_T.

2. A method of calculating the expected rate of return for a security as claimed in claim 1 wherein the risk factor comprises the volatility and/or higher moments of the expected return of the security.

3. A method for calculating the expected rate of return for a security issued by and/or referenced on a firm as claimed in claim 1 further comprising the steps of:

estimating the value of one or more variables in the multi-variate equation;

defining additional multi-variate equations representing relationships between some or all of the variables used in the multi-variate equation in claim 3;

equating one or more variables in said additional multi-variate equations with values specified by the user; and

solving all of the multi-variate equations to calculate the remaining unknown variables in the equation.

4. A method for calculating the expected rate of return for a security issued by and/or referenced on a firm as claimed in claim 1 wherein one of the variables in an additional multi-variate equation for which the user specifies a value comprises the volatility of the returns of one or more securities.

5. A method for calculating the expected rate of return for a security issued by and/or referenced on a firm as claimed in claim 1 wherein one or more of the variables in an additional multi-variate equation for which the user specifies a value represents the skewness, kurtosis and/or other higher moments of the returns of one or more securities.

6. A method for calculating the expected rate of return for a security issued by and/or referenced on a firm as claimed in claim 1 wherein one of the variables in an additional multi-variate equation for which the user specifies a value represents the correlation between the returns of a pair of securities.

7. A method for calculating the expected rate of return for a security issued by and/or referenced on a firm as claimed in claim 1 wherein one or more of the variables in an additional multi-variate equation for which the user specifies a value represents the correlation between the returns of a security and the returns of the total firm.

8. A method for pricing the debt and/or equity of a firm comprising the steps of:

calculating the expected rate of return for a security using the principles of the methodology as claimed in claim 1; and

defining a multi-variate equation representing the present values of the debt and/or equity based at least partly on B_{nT} and S_T.

9. A method for pricing the debt and/or equity of a firm as claimed in claim 8 further comprising the steps of:

estimating the value of one or more variables in the multi-variate equation;

defining additional multi-variate equations representing relationships between some or all of the variables used in the multi-variate equation in claim 8;

equating one or more variables in said additional multi-variate equations with values specified by the user; and

solving all of the multi-variate equations to calculate the remaining unknown variables in the equation.

10. A method for pricing the debt and/or equity of a firm as claimed in claim 9 wherein one of the variables in an additional multi-variate equation for which the user specifies a value comprises the volatility of the returns of one or more securities.

11. A method for pricing the debt and/or equity of a firm as claimed in claim 9 wherein one or more of the variables in an additional multi-variate equation for which the user specifies a value represents the skewness, kurtosis and/or other higher moments of the returns of one or more securities.

12. A method for pricing the debt and/or equity of a firm as claimed in claim 9 wherein one of the variables in an additional multi-variate equation for which the user specifies a value represents the correlation between the returns of a pair of securities.

13. A method for pricing the debt and/or equity of a firm as claimed in claim 9 wherein one or more of the variables in an additional multi-variate equation for which the user specifies a value represents the correlation between the returns of a security and the returns of the total firm.

14. A method of implementing an option-theoretic model of a firm, the method comprising the steps of:

selecting a plurality of input parameters, the parameters including a security specific risk premium in the expected rate of return for each security issued by, or referenced to, the firm;

defining relationships between the parameters; and

designating that for one or more priced risk factors in the model, the firm specific price per unit of the priced risk factor(s) is the same for two or more securities issued by, or referenced to, the firm.

15. A method of implementing an option-theoretic model of a firm as claimed in claim 14 further comprising the step(s) of determining values for unknown input parameters.

16. A method of implementing an option-theoretic model of a firm as claimed in claim 14 or claim 15 further comprising the step(s) of generating additional parameters from the model and solving the model so that the said parameters equal values specified by the user, where said parameters include:

the volatility of the returns of the common equity issued by the firm, or of securities referenced thereto;

the volatility of the returns of one or more debt securities issued by, or referenced to, the firm;

the skewness of the returns of one or more securities issued by, or referenced to, the firm;

the kurtosis of the returns of one or more securities issued by, or referenced to, the firm;

the higher statistical moments of the returns of one or more securities issued by, or referenced to, the firm;

the expected correlation between the returns of a pair of securities issued by, or referenced to, the firm;

the expected correlation between the returns of a security issued by, or referenced to, the firm and the returns of the total firm;

the expected probability of the firm defaulting on its debt;

the expected loss given default on a debt security issued by, or referenced to, the firm; and/or

the expected default loss (being the product of the expected probability of default and the expected loss given default) on a debt security issued by, or referenced to, the firm.

17. A method of implementing an option-theoretic model of a firm comprising the step(s) of generating additional parameters from the model and solving the model so that the said parameters equal values specified by the user, where said parameters include:

the volatility of the returns of one or more debt securities issued by, or referenced to, the firm;

the skewness of the returns of one or more securities issued by, or referenced to, the firm;

the kurtosis of the returns of one or more securities issued by, or referenced to, the firm;

the higher statistical moments of the returns of one or more securities issued by, or referenced to, the firm;

the expected correlation between the returns of a pair of securities issued by, or referenced to, the firm; and/or.

the expected correlation between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

18. A method of implementing an option-theoretic model of a firm as claimed in claim 14 wherein the priced risk factor comprises the volatility and/or higher moments of the expected return of one or more securities issued by, or referenced to, the firm and which risk factors have a firm specific price per unit of risk.

19. A method of implementing an option-theoretic model of a firm as claimed in claim 14 wherein one or more additional conditions are imposed so as to be able to solve the model for more than one unknown parameter.

20. A method of implementing an option-theoretic model of a firm as claimed in claim 19 wherein the model is solved so that one or more additional conditions imposed results in the model value for one or more additional parameters equalling a value specified by the user

21. A method of implementing an option-theoretic model of a firm as claimed in claim 18 wherein the firm has, or is treated as having, only a single class of zero coupon debt on issue and the model is fitted such that:

$$\frac{\ln\left(\frac{V_0 e^{r_V T}[1-N(d_1)]+B_0 e^{yT}N(d_2)}{B_0}\right)\Big/T - r}{\sigma_B} = \frac{\ln\left(\frac{V_0 e^{r_V T}N(d_1)-B_0 e^{yT}N(d_2)}{S_0}\right)\Big/T - r}{\sigma_S}$$

where:

S_n is the value of the (common) equity of the firm at time n

V_n is the value of the firm (the underlying assets) at time n. In this example the value of the firm is the sum of the values of the firm's debt (B) and equity (S)

X is the face value of the firm's debt (B), which is assumed to be a single zero-coupon bond, at maturity (i.e. the "exercise price")

T is the time to maturity (the expiry date) of the firm's debt in years

r_V is the rate of return on the firm's assets, per annum

y is the promised yield on the firm's debt, per annum

$$d_1 = \left(\left[\ln\left(V_0/X\right) + r_V T\right]\Big/\sigma_V\sqrt{T}\right) + (1/2)\left(\sigma_V\sqrt{T}\right)$$

$$d_2 = d_1 - \sigma_V\sqrt{T}$$

$N(\cdot)$ is the cumulative probability of the standard normal distribution with d_1 or d_2 as the upper limit

r is the risk free rate of return per annum

σ_V is the standard deviation of rates of return on the firm's assets per annum

σ_B is the standard deviation of rates of return on the firm's debt per annum

σ_S is the standard deviation of rates of return on the firm's equity per annum.

22. A method of implementing an option-theoretic model of a firm as claimed in any one of claims 16 to 21 wherein the formula for additional parameters generated from the model comprise:

instantaneous volatility:

$$\sigma_B = \sigma_V \frac{V_0}{B_0} e^{(r_V - r_B)T} [1 - N(d_1)]$$

$$\sigma_S = \sigma_V \frac{V_0}{S_0} e^{(r_V - r_S)T} N(d_1)$$

discrete time volatility and correlation:

$$\sigma_B = \sqrt{\ln\left(\frac{V_T^2 [1 - N(d_3)] e^{\sigma_V^2 T} + X^2 N(d_2)}{B_T^2}\right) \Big/ T}$$

$$\sigma_S = \sqrt{\ln\left(\frac{V_T^2 N(d_3) e^{\sigma_V^2 T} - 2V_T X N(d_1) + X^2 N(d_2)}{S_T^2}\right) \Big/ T}$$

$$\rho_{BS} = \frac{X - B_T}{B_T \sqrt{\left(e^{\sigma_S^2 T} - 1\right)\left(e^{\sigma_B^2 T} - 1\right)}}$$

$$\rho_{VB} = \frac{V_T [1 - N(d_3)] e^{\sigma_V^2 T} + X N(d_1) - B_T}{B_T \sqrt{\left(e^{\sigma_V^2 T} - 1\right)\left(e^{\sigma_B^2 T} - 1\right)}}$$

$$\rho_{VS} = \frac{V_T N(d_3) e^{\sigma_V^2 T} - X N(d_1) - S_T}{S_T \sqrt{\left(e^{\sigma_V^2 T} - 1\right)\left(e^{\sigma_S^2 T} - 1\right)}}$$

where the additional term is:

$$d_3 = d_1 + \sigma_V \sqrt{T}$$

23. A method of relating the volatility and correlation of returns of securities issued by, or referenced to, the firm to the expected default loss of one of the said securities (security *j*), wherein the annualised expected default loss on security *j* (EDL_j) is designated as:

$$EDL_j = \ln\left(\rho_{jk}\sqrt{\left(e^{\sigma_j^2 T}-1\right)\left(e^{\sigma_k^2 T}-1\right)}+1\right)\Big/T$$

where:

j is the class of the firm's debt or similar security issued by, or referenced to, the firm for which the expected default loss is being estimated

k is the class or classes of security issued by, or referenced to, the firm that rank immediately behind security j in terms of priority upon a liquidation or default event

T is the time horizon of interest to the user in years

σ_j is the standard deviation of rates of return, per annum, on the class of the firm's debt, or similar security issued by, or referenced to, the firm for which the expected default loss is being estimated

σ_k is the standard deviation of rates of return, per annum, on the portfolio comprising the class or classes of security issued by, or referenced to, the firm that are next most junior to j

ρ_{jk} is the correlation coefficient of the rates of return for j and k.

24. A method for determining the expected return of securities issued by, or referenced to, the firm wherein the expected default loss of another, debt-type, security (security j) issued by, or referenced to, the firm, (EDL_j) is utilised by:

determining the expected rate of return on security j (r_j) by reference to the promised yield on said security (y_j) and the expected default loss on said security where:

$$r_j = y_j - EDL_j$$

calculating the expected excess return for security j as equal to $r_j - r$, where r is the risk free rate of return;

calculating the exposure of each security to each priced risk factor (m);

calculating a price per unit of risk (λ_m) for each priced risk factor (m) in which λ_m is the same for two or more securities issued by the firm and such that the product of the risk exposures and prices per unit of risk for security j equals or approximates the expected excess return for security j (and similarly for any other security for which an estimate of the expected excess return is available);

designating λ_m's that relate to the volatility (or other higher moments) of the expected rate of return on securities and that are specific to the firm;

calculating the excess rate of return for all of the other securities, other than j, based at least partly on their expected exposure to each priced risk factor and the price per unit of risk (the λ_m's).

25. A method of calculating the expected rate of return for a security or securities as claimed in claim 24 (named security class or classes k) wherein the only priced risk factor comprises the volatility of returns and is implemented by:

designating the firm specific price of volatility risk (λ_σ), the volatility of returns for j (σ_j) and the risk free rate of return (r) as:

$$\lambda_\sigma = \frac{r_j - r}{\sigma_j}$$

designating the expected rate of return on security class or classes k (r_k) as:

$$r_k = r + \lambda_\sigma \sigma_k$$

designating, where security class or classes k are debt securities, the expected default loss on said securities by combining the promised yield on said securities (y_k) and their expected return (r_k) as follows:

$$EDL_k = y_k - r_k.$$

John Redmayne

By the authorised agents

AJ PARK

Per

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24 DEC 2003

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